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Formulation of a Combined Transportation and Inventory Optimization Model with Multiple Time Periods

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ABSTRACT
Most distribution network design models existing in the literature have focused on minimizing the costs of inventory and transportation. During the analysis of supply chain of currency management problem it is observed that the transportation of currency from various sources to various destinations and the required inventory to be maintained to meet the emerging demands requires formulation of a combined problem. This framework aims to support the coordination of inventory and transportation activities to properly manage the inventory profiles and currency flows between source locations and distribution centers. This paper considers a multi-period inventory and transportation model for a single commodity. The key contribution of this paper is, a mathematical programming formulation of transportation cum inventory problem is proposed and an algorithm for this new formulation as a multi period decision process is intended. A numerical example of currency transportation cum inventory is presented to illustrate the proposed algorithm.
Keywords: Transportation, and Inventory, Supply chain, Algorithm, multi- period.

I. INTRODUCTION

Here we briefly present transportation problem and inventory problem. We also present the importance of these two problems in the supply chain management and the motivation for the combined transportation and inventory optimization model along with the organization of the paper.

In a transportation problem (TP) there are multiple sources of supply units and multiple destination centers which demand certain quantity of single or multiple commodities from the supply centers. The transportation problem has the objective of minimizing the overall transportation cost by determining how much quantity from the respective sources to be sent to the various destinations keeping in view the available units at the source centers and the requirement at the destinations. The TP formulation was first given by F.L.Hitchcock (1941).. The source centers (S_i) may be production centers, plants, warehouses etc, whereas the destination centers (D_j) may be distributors, wholesale units, retail outlets etc.

In an inventory problem there may be one or multiple sources or production centers where single or multiple commodities in the form of raw materials or finished products are maintained for either production process or delivery to the various destinations. If the inventory of the required raw materials is not available in time the production process will be disturbed. On the other hand inadequate stock of finished products gives raise to reputation loss, orders loss etc.

The objective of the inventory problem is to determine the optimal quantity of the required items to be stocked which will minimize the overall inventory costs. The optimal quantity is called EOQ (Economic Order Quantity) and the optimal cost consists of purchasing cost, set up cost, holding cost and shortage cost.

A supply chain SC is a path from a production or supply source of a product or service to an end customer through a set of facilitating intermediate entities. These entities in a given market segment may be manufacturers, suppliers, transporters, wholesalers, retailers and agents. The objective of supply chain is to deliver a final product or service to a customer efficiently in time as per the specifications of the customer.

This has motivated us to look into the combined formulation of transportation and inventory in the supply chain. From the literature survey it is understood that much work is not done with multiple periods which minimizes the transportation and inventory costs together for single commodity with shortages. In fact, to the best of our knowledge there is no study at all which is incorporating the inventory costs in to the transportation model and extending it to the multiple periods. Our present work will concentrate on single commodity with multi periods.

This paper will combine the two approaches of transportation and inventory problem to develop a joint transportation cum inventory problem to determine the preferred compromised solution for multi period inventory cum transportation problem (MPICTP) for a single product, single stage with multiple suppliers, multiple destinations with deterministic demand for multiple periods. This approach leverages the advantages of considering both at a time to produce a powerful method to solve MPICTP. After observing the relevant models and important aspects presented in the previous literature a modified model is proposed. In the previous models, it is observed that the joint TP & IP model is formulated without taking either shortage costs or multiple time periods. Multiple time periods though considered, have not been taken continuously from one time period to another. With this proposed model period by period analysis is possible.

In section 2 Literature review of developments of transportation, inventory problems and the supply chain management of various supply chain networks are given. Problem description and an algorithm for the solution of the proposed formulation is given under section 3.Section 4 contains a numerical illustration by using the proposed Mathematical formulation.. The observations of the above proposed method and the numerical illustration are given under 5.The concluding remarks and the scope for further work are explained in 6. References are being given in the succeeding section 7.

II. RELATED WORK

The TP formulation was first given by F.L. Hitchcock (1941) and first inventory problem formulation(EOQ) was given by F.W. Harris in 1913.

In the literature development of the formulation of both inventory and transportation problems have taken independently. However several researchers have made attempts to combine both transportation and inventory for joint formulation. The first attempt in this direction was made by Brian Q. Rieksts, et al developed Optimal inventory policies with two modes of freight transportation European [1] (2008). Zhendong Pan,Jiafu Tang etc (2009) [2] Lihui Liu and Chunming Ye (2009) [3], Jae-Hun Kang Yeong-Dae Kim,[4] (2009), , Erhan Kutanoglu,Divi Lohiya (2008) presented an optimization based model to gain insights into the integrated inventory & transportation problem for a single- echelon, multi facility service parts logistics system with time based service level constraints.[5] Nikhil. A .Pujari, Trever. S Hale, Fazul Huq (2008) presented a continuous approximations procedure for determining inventory distribution schemas within supply chains. The model shows how inventory policy decisions directly impact expected transportation costs and provides a new method for setting stock levels that jointly minimizes inventory and transportation costs.[6]. Xiuli Wang,T.C.E Cheng [7] (2009) developed the heuristic algorithms considering both inventory and transportation costs simultaneously. Kadir Eritrogal (2007) suggested a Lagrangean decomposition based solution procedure for the problem of multi item single source ordering problem with transportation cost.[8] Simone Zanoni,Lucio Zavanella(2007) considered the problem of shipping products from a single vendor to a buyer with the objective of minimizing the sum of the inventory and transportation costs when the products are perishable. They have given a mixed integer linear programming formulation for it. [9]. K. Ertogral, M. Darwish, and M. Ben-Daya (2007) incorporated the transportation cost explicitly into the model and develop optimal solution procedures for solving the integrated models of joint vendor-buyer problem. [10]

A. Noorul Haq and G. Kannan.(2006) have proposed a two-echelon distribution-inventory supply chain model for the bread industry using genetic algorithm. Their paper presents an approach of optimizing the inventory level for a two-echelon supply chain by considering the distribution costs and various production related costs in meeting the customer demand.[11]. Linda van Norden et al dealt with a Multi-product lot-sizing with a transportation capacity reservation contract [12] (2005) , Cheng Liang Cen,Wen cheng Lee(2004) have given a multi product, multi stage & multi period scheduling model to deal with multiple incommensurable goals for a multi echelon supply chain network with uncertain market demands & product prices.[13]. Qiu Hong Zhao Shou-Yang Wangb, K.-K. Laic, Guo-Ping Xiaa. (2004) have presented a modified EOQ model including transportation cost.[14]. Linda K Nozick, Mark A Turnquist (2001) presented a modeling approach that provides an integrated view of inventory, transportation, serviced quality and the location of distribution centres.[15]

Wendy W Qu etc(1997) have proposed an integrated inventory transportation system with modified periodic policy for multiple products.[16] B.Q Rieksts et al [17] (2010) developed Two-stage inventory models with a bi-modal transportation cost, Nonihal Singh Dhakry et al [18] (2013) did research in Minimization of Inventory & Transportation Cost of an Industry – A supply chain optimization. They developed a new method by combining the transportation & inventory costs and minimizing them. Vivek K Agarwal et al [19] 2015 discussed an Inventory Model with Discrete Transportation cost and price break, Barry R Cobb [20] (2016) approached a method to an Inventory control for returnable transport itmes in a closed loop supply chain.

Looking at the previous works we have observed that the attempt to have a joint formulation of inventory and transportation originated with the extension of inventory problem.

III. PROPOSED TRANSPORTATION CUM INVENTORY MODEL

We present the mathematical programming formulation of a multi-period transportation cum inventory for a single item but for multi periods.

3.1 Assumptions

The various assumptions of the proposed model :

i). Transportation cost of each item to transport it from source locations to the destinations may vary across all the periods.

ii). Inventory-carrying cost of each item per period at all source locations and destinations may vary across multiple periods.

iii). Unit cost of the commodity at any source location is fixed, throughout the period of study at that plant.

iv). Transportation cost per product from the manufacturer to the source location is included in the unit commodity cost.

v). Daily demand of the demand centre is deterministic and the supply capacity of the selected supplier is limited. It is also assumed that the time taken to transport the item between the source location and the destination is homogenous and is not taken into consideration.

vi) Every item is considered as non perishable item for a certain number of periods.

vii) The centralized body which manages all the supply centers is concerned for over all optimization of inventory and transportation costs of these supply centers.

viii) Inventory and the transportation costs at the supply centers are only considered.

3.2 Notations:

M : Number of source locations of supply

N : Number of destination locations of demand

S_i : i^{th} source location point where i=1(1) m

D_j : j^{th} destination point where j= 1(1) n

TTC : Total transportation cost from all sources to destinations.

TIC : Total inventory cost at the source locations

S : Total number of time periods.

T^k : k^{th} time period, where k=1(1) s.

s_i^k : Total quantity of shortage of the commodity at i^{th} source S_i during time period T^k.

cs_i^k : Unit shortage cost at source Si during time period T^k.

h_i^k : Total quantity of a commodity held at i^{th} source S_i from the beginning of time period T^k

ch_i^k : Unit holding cost at source S_i during time period T^k

o_i^k : Ordering cost per order at S_i during k -th time period S_i.

N_i^k : Number of orders placed at i^{th} source S_i during time period T^k

a_i^k : Quantity of commodity available at S_i at the beginning of the time period T^k.

b_j^k : Quantity of commodity required at D_j at the beginning of the time period T^k

c_{ij} : Cost of transporting one unit of commodity from S_i to D_j.

x_{ij} : Quantity of commodity transported from S_i to D_j.

p_i^k : Unit cost of the commodity available at S_i during the time period T^k

Ta_i^k : Total quantity of commodity along with the carried forward number of units, if any available at S_i during T^k

Tb_j^k : Total quantity of commodity demanded along with the unfulfilled demand in the previous time period at D_j during T^k.

ub_j^k : Total quantity of unfulfilled demand at D_j during T^k

NC_i^k : Total number of units of commodity available at S_i during T^k

3.3 Mathematical Programming Formulation of Transportation cum Inventory Model

The objective is to

Min Z = W_1*TIC + W_2 *TTC =

$$w_1 * \left[\sum_{i=1}^{m} \sum_{k=1}^{s} [h_i^k * ch_i^k + s_i^k * cs_i^k + p_i^k * NC_i^k] + \sum_{k=1}^{s} \sum_{i=1}^{m} o_i^k * N_i^k \right] + w_2 * \left[\sum_{k=1}^{s} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}^k c_{ij}^k \right] \quad (1)$$

Where TIC = Total Holding Cost +Total Shortage Cost + Total Commodity Cost +Total Ordering Cost

TTC= Total Transportation Cost.

Subject to the Constraints

$$Ta_i^k = a_i^k + h_i^{k-1} \quad \text{for } i=1(1)\ m,\ k=1(1)\ s \quad \text{Where } h_i^0 = 0 \quad (2)$$

$$Tb_j^k = b_j^k + ub_j^{k-1} \quad \text{for } j=1(1)\, n,\ k=1(1)\, s \quad \text{where } ub_j^0 = 0$$
(3)

$$Ta_i^k - \sum_{j=1}^{n} x_{ij}^k = h_i^k \quad \text{for } i=1(1)\, m,\ k=1(1)\, s$$
(4)

$$Tb_j^k - \sum_{i=1}^{m} x_{ij}^k = ub_j^k \quad \text{for } j=1(1)\, n,\ k=1(1)\, s$$
(5)

$$\sum_{j=1}^{n} x_{ij}^k = Ta_i^k \quad \text{for } k=1(1)\, s$$
(6)

$$\sum_{i=1}^{m} x_{ij}^k = Tb_j^k \quad \text{for } j=1(1)\, n,\ k=1(1)\, s$$
(7)

$$\sum_{j=1}^{n} ub_j^k = \sum_{i=1}^{m} s_i^k \quad \text{for } k=1(1)\, s$$
(8)

$$h_i^k * s_i^k = 0 \quad \text{for } i=1(1)\, m,\ k=1(1)\, s$$
(9)

$$x_{ij} \in N \quad \text{for } i=1(1)\, m,\ j=1(n)\, 1$$
(10)

$$w_1 + w_2 = 1 \quad \text{for } 0 \le w_1 \le 1,\ 0 \le w_2 \le 1$$
(11)

On simplification, the k-th time period problem where k=1(1)s formulated results into:

Min Z=

$$\sum_{i=1}^{m} (h_i^k * ch_i^k + s_i^k * cs_i^k + p_i^k * NC_i^k + o_i^k * N_i^k) + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij}^k$$
(12)

Subject to the Constraints

$$Ta_i^k = a_i^k + h_i^{k-1} \quad \text{for } i=1(1)\, m,\ k=1(1)\, s \quad \text{Where } h_i^0 = 0$$
(13)

$$Tb_j^k = b_j^k + ub_j^{k-1} \quad \text{for } j=1(1)\, n,\ k=1(1)\, s \quad \text{Where } ub_j^0 = 0$$
(14)

$$Ta_i^k - \sum_{j=1}^{n} x_{ij}^k = h_i^k \quad \text{for } i=1(1)\, m,\ j=1(n)\, 1,\ k=1(1)\, (s-1$$
(15)

$$Tb_j^k - \sum_{i=1}^{m} x_{ij}^k = ub_j^k \quad \text{for } i=1(1)\, m,\ j=1(1)\, n,\ k=1(1)\, (s-1)$$
(16)

$$\sum_{j=1}^{n} ub_j^k = \sum_{i=1}^{m} s_i^k \quad \text{for } k=1(1)\, s$$
(17)

$$h_i^k * s_i^k = 0 \quad \text{for } i=1(1)\, m,\ k=1(1)\, s$$
(18)

$$x_{ij} \in N \quad \text{for } i=1(1)\, m,\ j=1(n)\, 1$$
(19)

$$w_1 + w_2 = 1 \quad \text{for } 0 \le w_1 \le 1,\ 0 \le w_2 \le 1$$
(20)

Equation 1 gives the objective which is the total cost of the supply chain network.

Constraint (2)-(3) represents the modified supply and demand values from the previous time period respectively. Eq (4)-(5) represents the units held, shortage respectively

Eq (6) describes the shortage at i^{th} supply centre is equal to the unfulfilled demand at j^{th} destination. Eq (7) describes either holding or shortage of units will occur at the source i.

The mathematical formulation for the proposed transportation cum inventory problem is first defined for the sum of k time periods and specially the k-th time period problem is defined as above. The objective of this problem is to minimize the transportation and inventory like holding, shortage, ordering and unit commodity costs. The corresponding algorithm for the solution of the above proposed transportation cum inventory problem is given below.

3.4 Algorithm for the solution of the proposed Transportation cum Inventory Problem:

Step1: Initialize time period k=1.

Step-2:Construct the transportation matrix by entering the supply amount a_i at i^{th} origin for i=1(1)m, demand b_j at j^{th} destination for 1(1) n and the unit costs c_{ij} in the $(i\ ,j)^{th}$ cell for i=1(1)m,j=1(1)n cells for the k^{th} time period k =1.

Step 3:-Find the initial basic feasible solution. We used the Vogel's approximation method after balancing the given transportation matrix.

Step 4:- Check for optimality of the obtained basic feasible solution and compute optimal solution by MODI's method.

Step 5:- Compute total transportation cost=Minimum transportation cost.

Identify the number of units carried forward or number of units of shortage and find the shortage cost or carrying cost.
Step-6: Compute total inventory cost=production cost + carrying cost + shortage cost.
Total cost = Total transportation cost + Total Inventory cost.
Step-7: Increment time period k=k+1 and go to step-2. Repeat the process until k=s.
Find the Cumulative total cost for the time periods k=1 (1)s
Stop if k>s

IV. VERIFICATION OF THE FORMULATION & ALGORITHM WITH A NUMERICAL EXAMPLE (STEP BY STEP EVALUATION)

A numerical example is given to illustrate the above algorithm. A general transportation problem is considered with 3 source locations and 4 destinations for 7 time periods. Capacities and demands are given for the respective source locations and destination points. The unit transportation cost is also given to transport 1 unit from any source to any destination. The number of commodities carried forward or number of units of shortage is identified and it will carry forward to the next time period. Initial basic feasible solution and the Optimum solutions are calculated by the Vogel's Approximation Method & MODI' s method respectively. The procedure will be continued for 7 time periods. Detailed illustration is given for k=1 and the same procedure is continued for the remaining time periods.

Time period k=1
Step -1: k=1
Step-2: Suppose the give transportation problem is given as follows.

	1	2	3	4	
I	6	3	5	4	22
II	5	9	2	7	15
III	5	7	8	6	6
	7	12	17	9	

The above considered transportation problem is not a balanced TP problem.(43≠45).To make it as a balanced add a 4^{th} source which is dummy. Modified problem is given below.

	1	2	3	4	
I	6	3	5	4	22
II	5	9	2	7	15
III	5	7	8	6	6
IV	0	0	0	0	2
	7	12	17	9	

Step-3: Initial basic feasible solution is identified by VAM.
$x_{12}=12, x_{13}=2, x_{14}=8, x_{23}=15, x_{31}=5, x_{34}=1, x_{41}=2$
Step-4: The optimal solution is found out by using MODI method which is given below
$x_{12}=12, x_{13}=1, x_{14}=9, x_{23}=15, x_{31}=6, x_{41}=1, x_{43}=1$
1 unit shortage in first destination& 1 unit shortage in destination 3 is identified .This will be carried to day-2.
Step-5: Total Transportation cost= 3*12+5*1+4*9+2*15+5*6+0*1+0*1=137/-
Step-6: Inventory cost =43*10/-+2*2/-+ 1*1= 430+4+1 =435/-
Total cost=Total transportation cost+ Total inventory cost. =137/- + 435/- = 572/-
Step-7: Increment the time period by 1. ie k =1+1 =2<7 proceed to the next day.

Formulation:-
The objective function is

$$\text{Min } Z = \sum_{i=1}^{3} \{h_i^1 * ch_i^1 + s_i^1 * cs_i^1 + p_i^1 * NC_i^1 + o_i^1 * N_i^1\} + \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}^1 x_{ij}^1$$

$$\Rightarrow \text{Min } Z = h_1^1 * ch_1^1 + s_1^1 * cs_1^1 + p_1^1 * NC_1^1 + o_1^1 * N_1^1 + h_2^1 * ch_2^1 + s_2^1 * cs_2^1 + p_2^1 * NC_2^1 + o_2^1 * N_2^1 + h_3^1 * ch_3^1 + s_3^1 * cs_3^1 + p_3^1 * NC_3^1 + o_3^1 * N_3^1 + c_{11}^1 * x_{11}^1 + c_{12}^1 * x_{12}^1 + c_{13}^1 * x_{13}^1 + c_{14}^1 * x_{14}^1 + c_{21}^1 * x_{21}^1 + c_{22}^1 * x_{22}^1 + c_{23}^1 * x_{23}^1 + c_{24}^1 * x_{24}^1 + c_{31}^1 * x_{31}^1 c_{32}^1 * x_{32}^1 + c_{33}^1 * x_{33}^1 + c_{34}^1 * x_{34}^1$$

$$= x_{11}^1 + 3 * x_{12}^1 + 5 * x_{13}^1 + 4x_{14}^1 + 5x_{21}^1 + 9x_{22}^1 + 2x_{23}^1 + 7x_{24}^1 + 5x_{31}^1 + 7x_{32}^1 + 8x_{33}^1 + 6x_{34}^1$$

$$= 0 + 1*3 + 10*22 + 0 + 0 + 10*15 + 0 + 1*3 + 10*6 + 6*6x_{11}^1 + 3x_{12}^1 + 5x_{13}^1 + 4x_{14}^1 + 5x_{21}^1 + 9x_{22}^1 + 2x_{23}^1 + 7x_{24}^1 + 5x_{31}^1 + 7x_{32}^1 + 8x_{33}^1 + 6x_{34}^1$$

After solving the problem by Vogel's approximation method and MODI's method the optimal solution is $x_{12}^1 = 12, x_{13}^1 = 1, x_{14}^1 = 9, x_{23}^1 = 15, x_{31}^1 = 6$ .Substitute these values in the above equation, then Min Z = 435/-+3*12/-+5*1/-+4*9/-+2*15/-+5*6/- =572/-
The no. of units of commodity available at the 3 source locations are as follows.
$a_1=22, a_2=15, a_3=6$
The no. of units of commodity required at the 4 destinations is as follows.
$b_1=7, b_2=12, b_3=17, b_4=9$.
The given problem is an unbalanced problem. The total availability=43

The total requirement is =45.
The total availability is less than that of the total requirement. So 2 units of shortage is identified. One dummy source location is added. We have $h_i^0 = 0$ and $ub_j^0 = 0$

$$Ta_1^1 = a_1^1 + h_1^0 = 22$$

$$Ta_2^1 = a_2^1 + h_2^0 = 15$$

$$Ta_3^1 = a_3^1 + h_3^0 = 6$$

$$Tb_1^1 = b_1^1 + ub_1^0 = 7$$

$$Tb_2^1 = b_2^1 + ub_2^0 = 12$$

$$Tb_3^1 = b_3^1 + ub_3^0 = 17$$

$$Tb_4^1 = b_4^1 + ub_4^0 = 9$$

$$a_i^1 - \sum_{j=1}^{4} x_{3j}^1 = h_i^1 \qquad i=1(1)3$$

$$b_j^1 - \sum_{i=1}^{3} x_{ij}^1 = ub_j^1 \qquad j=1(1)4$$

In this problem availability is less than that of the requirement. So no units will be carried forward to the next time period. But 2 units of shortage is identified. So these 2 units of unfulfilled demand will be carried forward to the next time period.

$$b_1^1 - \sum_{i=1}^{3} x_{i1}^1 = ub_1^1 \Rightarrow 7-(0+0+6) = 1$$

$$b_2^1 - \sum_{i=1}^{3} x_{i2}^1 = ub_2^1 \Rightarrow 12-(12+0+0) = 0$$

$$b_3^1 - \sum_{i=1}^{3} x_{i3}^1 = ub_3^1 \Rightarrow 17-(1+15+0) = 1$$

$$b_4^1 - \sum_{i=1}^{3} x_{i4}^1 = ub_4^1 \Rightarrow 9-(9+0+0) = 0$$

1 unit of shortage in D_1 & 1 unit of shortage in D_3 is identified. These units will be carried forward to the next time period to the corresponding destinations.

Time Period k=2:
The current time period inputs are given as

	1	2	3	4	
	1	3	4	2	6
	4	2	5	2	8
	3	6	1	5	10
	5	6	3	8	

After including the previous time periods carried forward inputs to the current inputs the problem is

	1	2	3	4	
I	1	3	4	2	6
II	4	2	5	2	8
III	3	6	1	5	10
	6	6	4	8	

After solving by Vogel's approximation & MODI's methods the optimal allotments are x_{14}=6, x_{22}=6,x_{24}=2,x_{31}=6,x_{33}=4

No shortage or held units are identified because the problem is a balanced transportation problem.
Total transportation cost =50/-
Total inventory cost= 24*8/-+1*1/- = 193/-
Total cost =Total transportation cost + Total Inventory cost = 50/- + 193/- =243/-
Cumulative total cost = Total cost of time period 1+ Total cost of time period 2
=572/-+243/- = 815/-
Increment the time period by 1 so k=2+1=3<7
Go to next time period.
Similarly the 3rd, 4th, 5th and 6th time periods inventory and transportation costs are calculated. Also the no. of units of shortage which is carried forward from the previous time period 6 is 2 units and it it in destination D_2.So to the existing current inputs of D_2 , 2 other units are added in the time period-7.

Time Period-7:
The current inputs for the fifth time period are as follows:

	1	2	3	4	
I	1	3	4	2	2
II	4	2	5	2	2
III	3	6	1	5	3
	1	2	1	1	

After balancing the problem by adding the dummy column and including the carried forward 2 units to D_2 the resultant input is as follows.

	1	2	3	4	
I	1	3	4	2	2
II	4	2	5	2	2
III	3	6	1	5	3
	1	4	1	1	

The optimum solution by MODI's method which is been obtained after finding the IBFS by Vogel's approximation method is given below.$x_{12}=1, x_{14}=1, x_{22}=2, x_{32}=1, x_{33}=1, x_{34}=1$
Total cost =Total transportation cost +total inventory cost
= Rs (137/-+50/-+14/-+21/-+21/-+24/-+19/-) + (435/-+193/-+129/-+95/-
+84/- +82/-+71/-) =Rs 1375/-
$K=7+1=8^{th}$ day > s
Stop, since k > s
The problem is a balanced transportation problem. No. of units required is equal to the no. of units demanded.
The consolidated results of the above problem are given below.

The original input table for 3 sources and 4 destinations:

Time period	Original inputs							Inputs after adding the carried forward or shortage no.of units from the previous time period.						
	Supply Si			Demand Dj				Supply Si			Demand Dj			
1	22	15	6	7	12	17	9	22	15	6	7	12	17	9
2	6	8	10	5	6	3	8	6	8	10	**6***	6	**4***	8
3	3	4	5	1	2	1	4	3	4	5	1	2	1	4
4	2	2	6	3	1	4	3	2	2	**10***	2	1	4	3
5	4	4	2	2	2	3	5	4	4	**5***	2	2	3	5
6	4	4	3	2	5	2	5	4	4	**4***	2	5	3	5
7	2	2	3	1	2	1	1	2	2	3	1	**4***	1	1

The Transportation & inventory input cost matrix:
For every time period shortage, holding, ordering & commodity costs are given. According to the requirement either shortage or holding cost is considered in every time period.

T^k	c_{11}^k	c_{12}^k	c_{13}^k	c_{14}^k	c_{21}^k	c_{22}^k	c_{23}^k	c_{24}^k	c_{31}^k	c_{32}^k	c_{33}^k	c_{34}^k	ch_i^k	cs_i^k	o_i^k	p_i^k
1	6	3	5	4	5	9	2	7	5	7	8	6	2	2	1	10
2	1	3	4	2	4	2	5	2	3	6	1	5	1	1	1	8
3	1	3	4	2	4	2	5	2	3	6	1	5	2	1	1	10
4	1	3	4	2	4	2	5	2	3	6	1	5	1	1	2	9
5	1	3	4	2	4	2	5	2	3	6	1	5	3	1	1	8
6	1	3	4	2	4	2	5	2	3	6	1	5	3	2	1	7
7	1	3	4	2	4	2	5	2	3	6	1	5	2	2	1	10

Optimal solutions for the 7 time periods which includes the minimized transportation and inventory costs.

T^k	x_{11}^k	x_{12}^k	x_{13}^k	x_{14}^k	x_{21}^k	x_{22}^k	x_{23}^k	x_{24}^k	x_{31}^k	x_{32}^k	x_{33}^k	x_{34}^k	h_i^k	s_i^k	N_i^k	TTC	TIC	MinZ
1	*	12	1	9	*	*	15	*	6	*	*	*	*	2	1	137	435	572
2	*	*	*	6	*	6	*	2	6	*	4	*	*	*	1	50	193	243
3	1	*	*	2	*	2	*	2	*	*	1	*	4	*	1	14	129	143
4	0	*	*	2	*	1	*	1	3	*	4	*	1	*	1	21	95	116
5	1	*	*	3	*	2	*	2	1	*	3	*	*	1	1	21	84	105
6	0	*	*	4	*	3	*	1	2	*	2	*	2	*	1	24	82	106
7	*	1	*	1	*	2	*	*	1	1	1	*	*	*	1	19	71	90

V. Concluding remarks

1 Here a mathematical programming formulation combining transportation and inventory problem is given. 2. This formulation considers giving priority to either transportation cost or inventory cost however in reality the transportation cost overshoots inventory cost. 3. From the proposed formulation a decision maker can get the information related to Inventory costs (shortage costs & holding costs) and Transportation costs separately in each time period. 4. The number of commodities that are carried forward and shortage of the number of items in each stage can also be known by the decision maker. 5. This is a multi period formulation for a single objective and can be extended to multi objectives and for multi products.

In the above numerical example, all the types of transportation problems like balanced, unbalanced, degeneracy in the initial basic feasible solution and degeneracy in the optimum solution are observed. Also for some time periods, carried forward commodities and for some other time periods, shortage of commodities is observed. This proposed formulation will work though the cost per unit C_{ij} values is different for different time periods. This problem is solved again by a familiar linear programming technique (penalty method) and the results are compared. It is concluded that the optimal solution is same for both the above said methods.

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